Coefficient Tests for Stability

First- and Second-Order Systems

The stability of first- and second-order systems can be determined by inspection of the coefficients of the characteristic polynomial. Both systems are stable, with all roots in the left half of the complex plane, if and only if all polynomial coefficients have the same algebraic sign.

For first-order polynomials, the proof is trivial. Now consider the following second-order polynomial with leading coefficient equal to one.

$$s^2 + a_1 s + a_0 = 0$$

Applying the quadratic formula,

$$s = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_0}}{2}$$

$$= -\frac{a_1}{2} \pm \frac{\sqrt{a_1^2 - 4a_0}}{2}$$

Clearly $a_1 > 0$ is required. In addition, for real and distinct roots,

$$-a_1 + \sqrt{a_1^2 - 4a_0} < 0$$

$$\sqrt{a_1^2 - 4a_0} < a_1$$

$$a_1^2 - 4a_0 < a_1^2$$

$$-4a_0 < 0$$

$$\underline{\underline{a_0 > 0}} \leftarrow$$

Higher-Order Systems

For higher-order polynomials representing higher-order systems, the algebraic signs of the polynomial coefficients may or may not yield information as to stability. The following two conditions do result in conclusions about polynomial roots.

1. Differing algebraic signs - At least one RHP root.
2. Zero-valued coefficients - Imaginary axis roots or RHP roots or both.

Examples:

$$s^5 + 4s^4 - 3s^3 + s^2 + 7s + 10 = 0$$

roots =
- $0.9098 + j\,0.7826$
- $0.9098 - j\,0.7826$
- -1.3689
- $-0.6254 + j\,0.7895$
- $-0.6254 - j\,0.7895$

$$s^4 + 3s^3 + 2s + 6 = 0$$

roots =
- -3.0000
- $0.6300 + j\,1.0911$
- $0.6300 - j\,1.0911$
- -1.2599
